Chapter Three Homework

Homework Question 4

Imagine that the LIGO experiment used a normal Michelson interferometer that had no Fabry-Perot cavities in it. The arms begin with the same length, call it L, but then something happens to cause one of them to change by some amount ΔL. How does this affect the relative phase of the two beams when they exit the beam splitter on their way out of the interferometer if the light has frequency f and wavelength λ? What is your answer numerically for values appropriate for LIGO, where $\lambda = 1064\,\mathrm{nm}$ and $\mathrm{L} = 4\,\mathrm{km}$ and $h \sim 10^{-21}$?

Homework Question 5

Repeat the previous problem, but now assume that each arm of the interferometer contains a Fabry-Perot cavity that causes the laser beams to traverse the arms twice before exiting. How will this affect the phase difference? The number of times that each FP cavity is traversed is called the *finesse* of the cavity. So for this example we would say that the finesse of the cavities is 2. Devise a relation for the phase difference of the two beams upon exit and the finesse of the FP cavities they traverse?

Homework Question 6

In this exercise you will explore the interference pattern of co-added waves. Use a spreadsheet (or other software if you wish) to compute the sine function from zero to 6π. Plot the function.

Next, compute another sine function with the same frequency (or wavenumber) over the same interval, but shift its phase. Try various shifts between zero and 2π. Add the two sine waves together and plot their sum. Notice how the sum changes as the relative phase of the two sine functions changes.

At what phase do the waves exactly cancel? At what phase do they maximally reinforce one another?

This type of interactive exercise might be useful for students who are just learning about waves and their addition. By putting the value of the phase offset in one cell of a spreadsheet and referencing that cell, it is easy to compute the sum for different offsets and immediately see the plots change. If you like, you can have other cells that contain the wave-numbers for each sine function. That would let you easily see how changing each wavenumber affects the sum.

Other software can do this too. For example, Python, Mathematica and Matlab would also be able to perform this task without much difficulty, but they are not as widely available. Or if you have a Mac, the included graphing calculator software (called Grapher) does a good job with this, making somewhat neater plots. The tradeoff is that you cannot see the computed numerical values for the functions the way you do with a spreadsheet. Seeing these numbers might be instructive for students.