Chapter Four Homework

Homework Question 7

Assuming that the gas inside the vacuum chambers has the same composition as the outside air, estimate the following. a) The particle density inside the chambers. b) The total number of particles inside one of the arms. c) The typical distance between particles. d) The rate at which particles impact each square meter of the inside of the vacuum chamber. Each of the arms is 4 km long and 1.2 m in diameter. The temperature inside the chamber is around room temperature, $\sim 300\,\mathrm{K}$.

Homework Question 8

While the LIGO test masses reside in a vacuum environment, it is not a particularly cold one. The temperature inside is whatever the ambient room temperature is in the lab, around 300 K. Use the Equipartition Theorem to estimate the motion of the 40 kg test masses. You can use the last stage of the pendulum if you like, ignoring the others.

Homework Question 9

In addition to transverse kinetic energy, each pendulum has thermal energy partitioned into the strings that support it. The vibrational energy in a string under tension is given by the following equation.

$$E_n = A_n^2 \frac{\pi^2 \tau}{4L} n^2$$

Here, A_n is the amplitude of the n^{th} vibrational mode of the string, τ is the tension in the string, and L is the length of the string. From Figure 4.1 you can see that each test mass is suspended by two pairs of silica fibers, so that each fiber supports one fourth the weight of each mass. Assuming that the fibers are 60 cm in length, compute the amplitude of the fundamental (n=1) vibrational mode. How does this amplitude compare to the size scale you found in the previous exercise? Do you think that the assumption of a vibrating string with two "fixed" ends is a good one? The frequency of such a string is given as follows.

$$f_n = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}}$$

The mass per unit length of each fiber is denoted by μ. The density of fused silica is $\rho = 2.2\,\mathrm{g\,cm^{-3}}$, and each fiber has a diameter of 0.4 mm. Estimate the vibrational frequency of the fibers. Keep this frequency in mind. It will prove relevant later on in Section 5.

Homework Question 10

The wavelength of the LIGO laser is 1064 nm and its power inside the Fabry-Perot cavities is 850 W. In this problem you will use this information to explore the effect of statistical fluctuations of the laser light on the motion of the 40 kg test masses. This *shot noise* is caused by Poisson fluctuations on the numbers of photons. Use a characteristic time scale of 100 Hz, typical for the frequencies of the gravitational waves LIGO can detect, and complete the following steps.

a) How much laser energy hits the test mass in one time scale?

b) How many photons is this?

c) What is the Poisson fluctuation on this number?

d) What fluctuation in photon momentum corresponds to this photon fluctuation?

e) What fluctuation in the momentum of the test mass does this impart?

f) What fluctuation of the speed of the test mass corresponds to this?

g) How far does the test mass move during this timescale?