

Astrophysical Sources of Gravitational Waves

August 11, 2015

1 Overview

In the previous units, we have reviewed the properties of electromagnetic radiation, and have shown how accelerating charges produce transverse electromagnetic waves. We have also investigated simplified models that show how gravitational waves are emitted as “ripples in spacetime” by accelerating masses. Gravitational waves are also transverse waves that travel at the speed of light. General relativity allows us to predict the strength and frequency of these waves, just as Maxwell’s equations can be used to predict the strength and frequency of electromagnetic waves emitted from various types of objects and processes. However, electromagnetism is a much stronger force than gravity and it is easy to detect a wide spectrum of electromagnetic waves from common sources (not just astronomical bodies). Because gravity is by far the weakest of the four forces, gravitational waves have thus far not been directly detected. A strong gravitational wave will produce displacements on the order of 10^{-18} meters - this is ~ 1000 times smaller than the diameter of a proton! And it takes immensely massive astronomical bodies to create the waves that we hope to detect. And while electromagnetic waves are primarily due to time-varying dipoles (either accelerating charges or currents), gravitational waves require time-varying quadrupoles, such as orbiting masses, or aspherical rotating or exploding bodies. Another similarity between electromagnetic and gravitational waves is the existence of two independent states of polarization. For gravitational waves these states are typically designated as h_+ and h_\times . See Figure 1 for an illustration of the effects on a ring of test masses as the two different polarizations of gravitational waves pass.

As you saw in the *Big Ideas* chapter on General Relativity, indirect evidence for gravitational radiation was first reported by Hulse and Taylor (1975) for the PSR 1916+13 binary system. Their Nobel-prize winning measurement of the decay of the binary orbit as

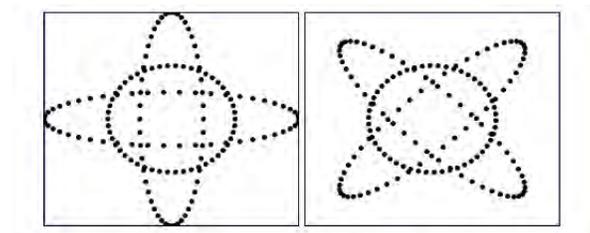


Figure 1: The effect on proper separations of particles in a circular ring in the (x, y) -plane due to a h_+ wave traveling in the z -direction is shown in the left-hand panel, and due to a h_\times wave is shown in the right-hand panel. The ring continuously gets deformed into one of the ellipses and back during the first half of a gravitational wave period and gets deformed into the other ellipse and back during the next half. In contrast to electromagnetic waves, the angle between the two polarization states is $\pi/4$ rather than $\pi/2$.^[1]

a function of time agreed with that predicted due to the emission of gravitational radiation. Although the gravitational wave frequency predicted of $\sim 70 \mu\text{Hz}$ is too low to be detected by current ground-based gravitational wave interferometers, there are other astronomical binaries that may be closer to the end point of coalescence, and that would create much higher-frequency signals. Additional likely sources of gravitational waves include impulsive events, such as asymmetric supernovae or gamma-ray bursts, as well as continuous sources, such as asymmetric rotating neutron stars. In the sections below, we will describe these source classes, and will also summarize the predictions for detections in the current era.

Homework 1

What is the orbital period of the Hulse-Taylor pulsar, PSR 1916+13, given that the predicted gravitational wave frequency is $\sim 70 \mu\text{Hz}$? Explain.

2 Gravitational Wave Detectors

Directly detecting gravitational waves is the scientific goal for ground-based gravitational wave observatories located around the globe, including LIGO, the Laser Interferometer Gravitational-wave Observatory. LIGO has two 4 km-long interferometers: one is located in Hanford, Washington (Figure 2) and the other in Livingston, Louisiana (Figure 3).



Figure 2: LIGO Hanford Observatory in Washington



Figure 3: LIGO Livingston Observatory in Louisiana

Recent upgrades to both of these facilities have increased LIGO's sensitivity by more than a factor of 100, and if the predictions are correct, the first gravitational waves may be directly detected within the next two years. The amazing hardware that has been installed in LIGO to detect such tiny displacements will be the subject of next year's online course, *LIGO Technology*. For further information about the LIGO detectors, see <http://www.ligo.org>.

Additional information will be obtained starting in 2016 through observations with the 3-km interferometer, Virgo (near Pisa, Italy, see Figure 4). A fourth, smaller observatory, GEO, is operational in Germany. KAGRA, an underground observatory is under construction in Japan, and plans are underway to construct a southern-hemisphere observatory, LIGO-India. Installation of an observatory in the southern hemisphere would allow better localization of sources on the sky, which will help with identifications of electromagnetic counterparts.



Figure 4: The European gravitational wave observatory, Virgo, near Pisa, Italy

For further information about Virgo, see <http://www.ego-gw.it/public/virgo/virgo.aspx>. For further information about the GEO detectors, see <http://www.geo600.org/>. For further information about KAGRA (in English), see <http://gwcenter.icrr.u-tokyo.ac.jp/en/>.

Space-based observations of gravitational waves have been contemplated for more than

20 years. Recently, technology has advanced to the point that a launch is near for the “LISA pathfinder” mission. LISA (Laser Interferometer Space Antenna) was originally a joint NASA-European Space Agency (ESA) mission, but due to funding changes, the pathfinder mission has been built and will be launched by ESA in late 2015. The main experiment on board LISA Pathfinder is the LISA Technology Package (LTP) which will accurately control and measure the positions of two test masses in near-perfect gravitational free fall. The masses are made of gold and platinum, and hence are reflective of laser light and can act as mirrors for a space-based interferometer. A smaller instrument package, the Disturbance Reduction System, was contributed by NASA to help control the spacecraft. The original LISA concept was for three spacecraft to fly in formation with separation distances of $\sim 5 \times 10^6$ km, which would allow measurements of gravitational waves at much lower frequencies (1 - 30 mHz) than ground-based interferometers (with typical lengths of 3-4 km, that aim to detect gravitational waves in the 50 - 2000 Hz band). Figure 5 shows the science module for the LISA Pathfinder mission undergoing testing in April 2015. For more information about the LISA Pathfinder mission, see <http://sci.esa.int/lisa-pathfinder/>.



Figure 5: The LISA Pathfinder science module in a clean room in Germany. The large central cylinder is the core assembly for the LISA Technology Package. One of two (gold coloured) colloidal micro-Newton thrusters, provided by NASA, can be seen on the left side of the spacecraft. Credit: ESA – U. Ragnit

Recently, the National Science Foundation has established a Physics Frontier Center to use precise pulsar timing as a means to detect gravitational waves. The North American Nanohertz Observatory for Gravitational Waves (NANOGrav) was established in 2007 by radio astronomers who time the emission from an array of nature’s most precise clocks - the “millisecond pulsars.” By measuring nanosecond changes in the timing signals from these pulsars over many years, NANOGrav will be sensitive to gravitational waves in the

nanoHz frequency band. These extremely long wavelength gravitational waves are primarily expected to arise from a stochastic background of merging supermassive black holes, individual inspiral and merger events of supermassive black holes in distant galaxies, and possibly cosmic strings and inflationary gravitational waves. There are over 30 millisecond pulsars distributed throughout our galaxy, and the pulsations from these pulsars will be measured by the world's largest and most sensitive radio telescopes. Figure 6 shows the Arecibo telescope in Puerto Rico, which is the world's largest radio dish. Arecibo and the Green Bank Telescope in West Virginia are each used to time 19 millisecond pulsars in the array while searching for additional candidate pulsars to increase the sample. A new, even larger telescope FAST ([Five-hundred-meter Aperture Spherical Telescope](#)) is under construction in China and may soon be contributing to these types of observations. For more information about the NANOGrav project, see <http://nanograv.org>.



Figure 6: The 305-meter dish of the William E. Gordon Telescope. Image credit: NAIC

The longest gravitational waves that are predicted emanate from the rapid period of inflation that followed the Big Bang. Inflationary theory predicts that the gravitational waves should have left an imprint on the cosmic microwave background that is proportional to the expansion rate during inflation, and that this imprint should be measurable in the polarization of the electromagnetic microwave background signals. In March 2014, considerable media coverage was generated when the BICEP team announced preliminary evidence for the discovery of these polarization signals. (Read [Sean Carroll's initial column](#) for a good summary of the physics and his [follow-up column](#) that explains what was reported by the BICEP team.) However, by September 2014 [further analysis of data from ESA's Planck mission](#) indicated that the results were overstated and most likely due to scattering by dust. To read more about the physics of the CMB polarization process, see

<http://cosmology.berkeley.edu/~yuki/CMBpol/CMBpol.htm>.

Just as different astrophysical sources generate different wavelengths across the electromagnetic spectrum, different physical systems are expected to generate different wavelengths of gravitational waves, which will be observed by different types of observatories. Figure 7 illustrates the expected strains (a measure of the strength of the gravitational wave, see Equation 5), wavelength bands, types of sources and the observatories that are now being built to probe the universe in this entirely new way.

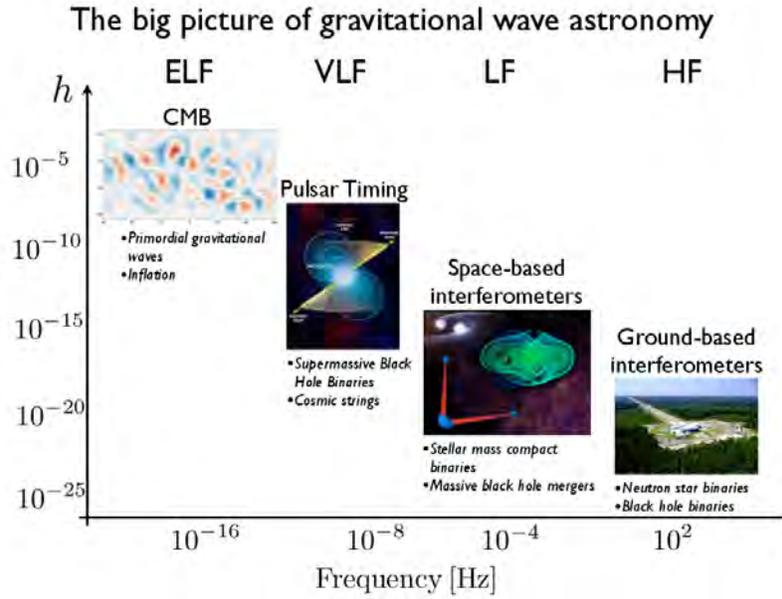


Figure 7: Gravitational waves (GWs) span many orders of magnitude in frequency, from the Hubble-length primordial waves that leave their imprint on the cosmic microwave background (CMB), to the gravitational waves with periods of years detectable by pulsar timing arrays (PTAs) like NANOGrav, the hour-long period waves detectable by space-based instruments such as LISA, and the millisecond period waves detectable by ground-based interferometers like LIGO and Virgo. Credit: NANOGrav

3 Coalescing Binaries

Compact binaries are defined as systems that include either two neutron stars, two stellar-mass black holes, or one of each. These systems are formed when a binary system consisting

of two massive stars evolves: the more massive star ends its life with a supernova explosion that does not disrupt the binary, and forms a neutron star. Matter from the less massive star is accreted by the neutron star, increasing its spin rate, in a process known as “recycling.” The less massive star also eventually goes supernova, producing a second neutron star in the binary. If the orientation of the system is favorable (and the neutron star magnetic fields are strong enough), pulses may be observed from one or more of the neutron stars in the binary system. Figure 8 shows an artist’s conception of merging neutron stars.

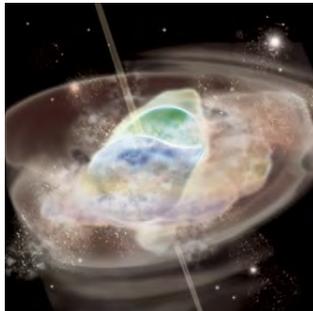


Figure 8: Artist’s conception of merging neutron stars. Credit:NASA/SSU/A. Simonnet

The Hulse-Taylor binary pulsar system is comprised of two neutron stars, with approximately equal masses, $m \sim 1.4M_{\odot}$. In this system, only one of the neutron stars is observed to emit pulsations. Others in this category include B1534+12, J1829+2456, and B2127+11. The “double pulsar” system PSR J0737-3039, discovered in 2003 by Marta Burgay and her colleagues at Parkes Observatory [3], is the only system thus far discovered that consists of two neutron stars which are both emitting observable pulsations. This system also has the observational advantages that it is about 10 times closer to the Earth and will coalesce about 3.5 times faster than the Hulse-Taylor pulsar. As we will see below, these facts are important inputs to the calculations of expected LIGO detections from coalescing binaries.

Modeling these systems is usually done using Newtonian approximations to the gravitational field, while treating each neutron star as a point mass. In this case, the orbital energy of the system is given by:

$$E_{orb} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{r} = \frac{G\mu M}{2r} \quad (1)$$

where r is the separation between the two masses, m_1 and m_2 , $M = m_1 + m_2$ is the total mass, $\mu = m_1m_2/M$ is the reduced mass, and G is the gravitational constant. Using the Newtonian approximation, we find [4] that the luminosity \mathcal{L} of gravitational waves emitted

by the coalescing system will be:

$$\mathcal{L} = -\frac{32 G}{5 c^5} \mu^2 R^4 \Omega^6 = -\frac{dE_{orb}}{dt} \quad (2)$$

where $\Omega = \sqrt{\frac{GM}{R^3}}$ is the orbital frequency. We can rewrite the time derivative of E_{orb} in order to find the equation to integrate that results in the prediction for the orbital separation as a function of time $r(t)$:

$$\frac{dE_{orb}}{dt} = \frac{dE_{orb}}{dr} \frac{dr}{dt} = -\mathcal{L} \quad (3)$$

$$r(t) = \left(\frac{256 G^3}{5 c^5} M^2 \mu \right)^{\frac{1}{4}} (t_c - t)^{\frac{1}{4}} \quad (4)$$

where t_c is the coalescence time (the time when the separation of the two masses goes to zero).

In order to estimate the rate at which LIGO (when it reaches its currently planned design sensitivity) can expect to detect coalescing binaries, we need to consider the the estimated merger rate of binary neutron stars in our galaxy and multiply this by the number of galaxies within LIGO's detection range. The estimated merger rate is found by dividing the number of binary neutron star systems in our galaxy by the time it takes them to merge. Since the double pulsar system has a time to coalescence about 1/3 that of the Hulse-Taylor pulsar, this has a positive effect on the predictions for LIGO detections. The estimate of the number of binary neutron star systems in our galaxy is also improved by the detection of weaker radio emission from the double pulsar system, compared to the Hulse-Taylor pulsar. Detecting weaker radio emission from a system that is ten times closer to Earth implies the existence of many more similar systems that have not yet been seen. Burgay et al. (2003)[3] estimated that the detection rate of coalescing binaries with LIGO (prior to its recent upgrade) could be as high as 1 every 1-2 years at the 95% confidence level. The fact that no coalescing binaries were seen with LIGO in the pre-upgrade era is consistent with these estimates. More refined estimates for upgraded interferometers can be found in Abadie et al. (2010)[7], a paper that includes authors from both the LIGO and Virgo collaborations.

Improvements in LIGO instrumentation, now being brought online, and in Virgo (scheduled for 2016) are usually characterized by predicting the volume of space within which a particular class of sources detections are expected to occur. For example, with instrumentation available in 2002, LIGO could only hope to detect neutron star binary coalescences

within the Milky Way galaxy. By 2003, the reach had expanded to our local group (including M31), and by 2005 to the Virgo cluster (20 Mpc). When the current instrumentation reaches its design sensitivity (probably by 2017-2018), the reach will expand to ~ 200 Mpc in general and up to 450 Mpc for an optimally-oriented binary. The change in detection volume from initial to the current LIGO hardware (“Advanced” LIGO) is illustrated in Figure 9.

Of course, these detection volumes depend critically on the assumed strength of the gravitational waves. For a typical double neutron star binary (each with mass equal to $1.4M_{\odot}$), in a very close orbit of 20 km, with an orbital frequency of 400 Hz (and hence gravitational waves with a frequency of 800 Hz), and using our Newtonian, point-mass approximation, the dimensionless strain h is given by [5]:

$$h = \Delta L/L \approx \frac{10^{-21}}{(r/15 \text{ Mpc})} \quad (5)$$

Note that measuring strains of this magnitude was marginally within the sensitivity range of initial LIGO, but is well within the design sensitivity of LIGO’s new instrumentation, as shown in Figure 10. However, it should also be realized that strains this large will only occur near the final endpoint of the coalescent process. For detailed information on recently published limits to detections in various coalescence scenarios, see the review by Riles (2013)[6].

The characteristic pattern of strain that will be emitted requires numerical calculations for the strong field (and hence non-Newtonian) gravity at the endpoint. A typical inspiral gravitational wave form is shown in Figure 11.

Homework 2

Listen to the gravitational waveform from [two merging neutron stars](#) and to the same signal [in a noisy environment](#). Then navigate to Prof. Scott Hughes’ [website](#) and compare and contrast the “sounds” of at least two different types of binary system coalescence events.

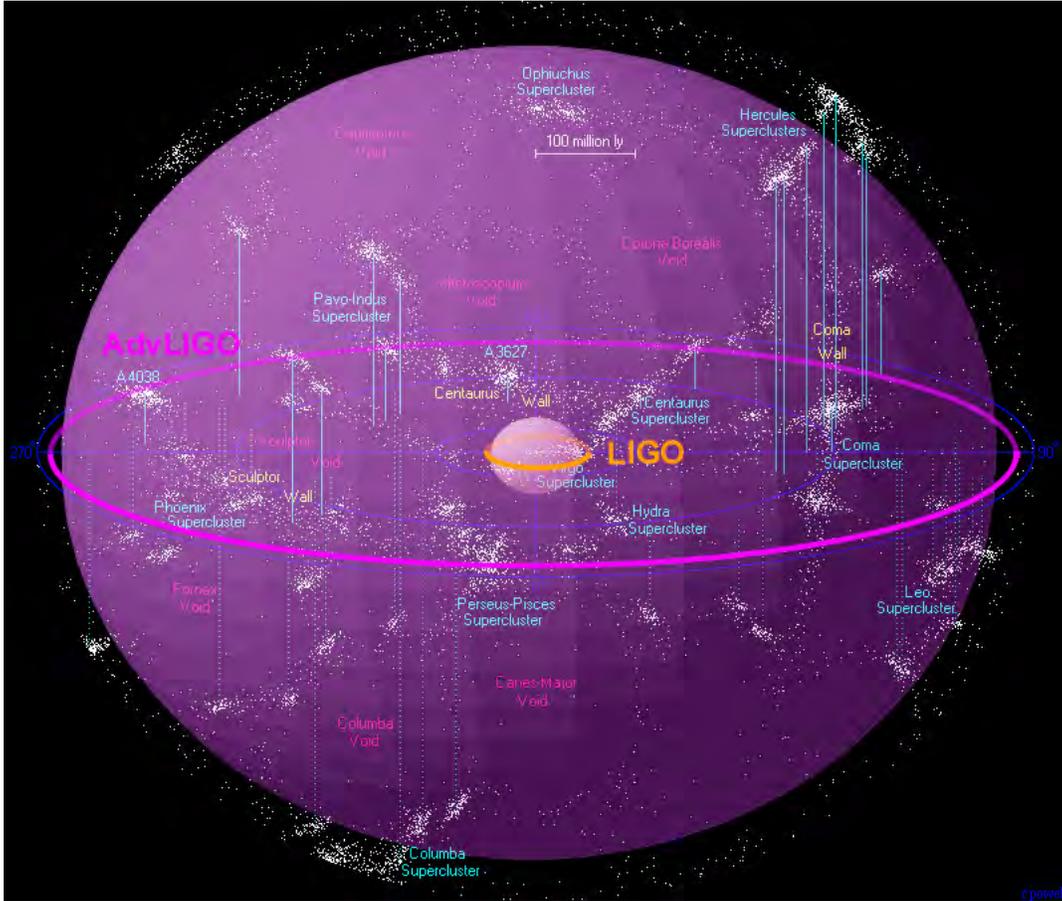


Figure 9: The volume of space within which neutron star binary coalescence events should be detectable for initial LIGO observations (orange ring and light purple sphere) compared to the much larger volume predicted for Advanced LIGO (darker purple ring and sphere, about 200 Mpc diameter). Credit: Beverly Berger for the LIGO overlays, Richard Powell for the astronomical map at <http://www.atlastoftheuniverse.com>.

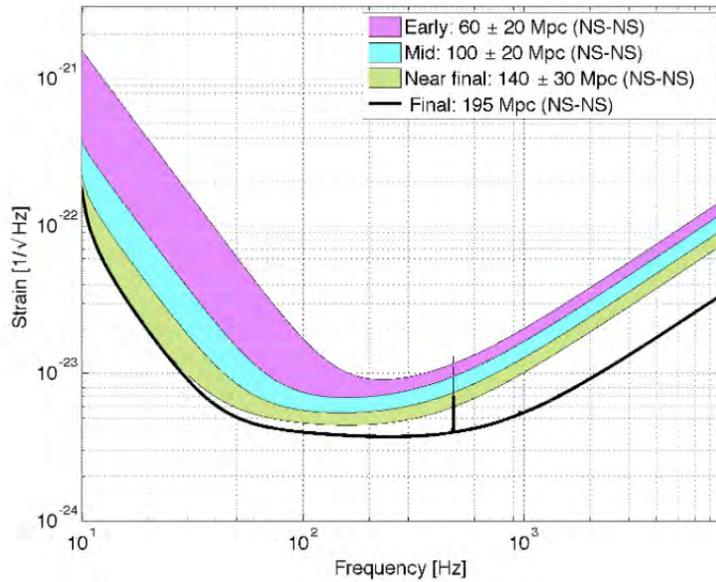


Figure 10: Predicted sensitivity curves for the upgraded LIGO facilities for neutron star binary coalescence events. The sensitivity is predicted to reach to ~ 200 Mpc by 2017-2018. The different curves represent phases in the commissioning of the instrumentation, beginning with the first observing run in the fall of 2015 (purple shaded area).

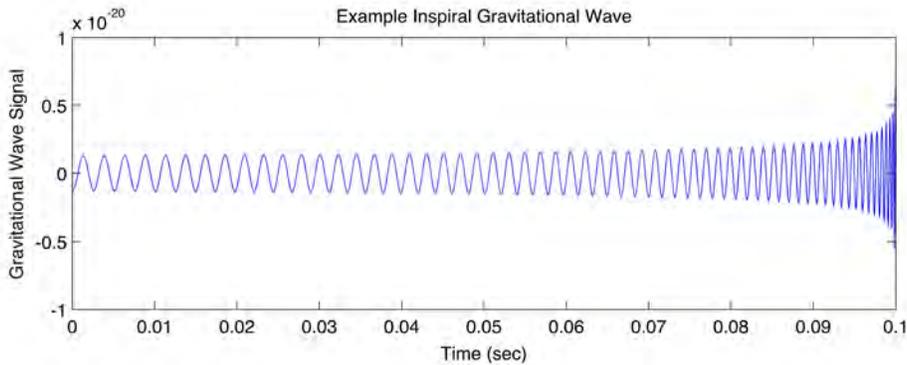


Figure 11: Example gravitational wave form from a coalescing binary. The gravitational wave signal is the strain h . Credit: A. Stuver

4 Continuous Wave Sources

Rotating neutron stars can be sources of continuously emitted, monochromatic gravitational waves if they are not axisymmetric. A typical expected wave form is shown in Figure 12. These neutron stars will be potentially detectable by LIGO if they are spinning rapidly enough that twice the rotation frequency is within LIGO's spectral band (see Figure 10). There are many well studied neutron stars in our Milky Way galaxy with rotation rates in the 10 - 1000 Hz range, including radio and gamma-ray emitting pulsars (typically isolated objects), X-ray pulsars (typically in binary systems with mass-donating companions), and some quasi-periodic X-ray emitters (also typically in accreting binary systems). Many of these potential sources have well-measured pulsation or inferred spin rates, and some also have well-determined binary parameters. Knowledge of these parameters helps narrow down the parameter space that needs to be searched for potential gravitational wave signals.

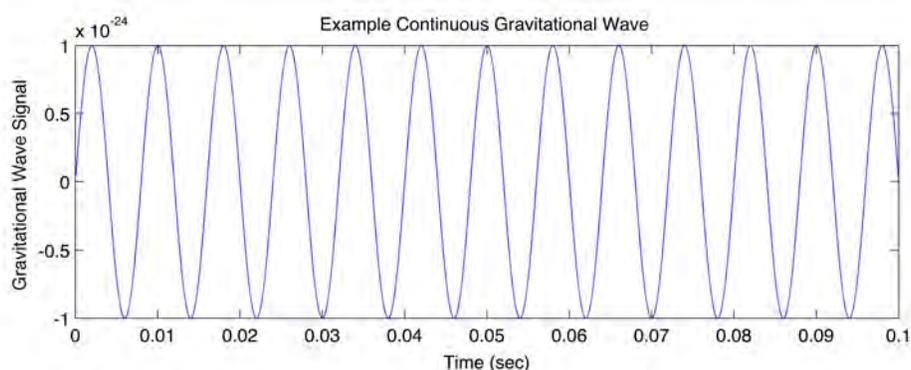


Figure 12: Example gravitational wave form from a continuous wave source. The gravitational wave signal is the strain h . Credit: A. Stuver

In order to predict the approximate strength of the expected gravitational wave signals, we need to make some assumptions about the extent to which a neutron star will deviate from spherical symmetry. Following Riles (2013)[6], we parameterize the quadrupole asymmetry by its ellipticity:

$$\epsilon \equiv \frac{I_{xx} - I_{yy}}{I_{zz}} \quad (6)$$

where the terms I_{xx} , I_{yy} and I_{zz} are the diagonal Cartesian components of the moment of

inertia tensor, and the star is assumed to be (approximately) spinning around its z -axis. In the optimal case, the spin-axis will be pointed towards Earth, and the expected strain will be given by[6]:

$$h_o = \frac{4\pi G I_{zz} f_{GW}^2}{c^4 r} \epsilon = (1.1 \times 10^{-24}) \left(\frac{I_{zz}}{I_o} \right) \left(\frac{f_{GW}}{1 \text{ kHz}} \right)^2 \left(\frac{1 \text{ kpc}}{r} \right) \left(\frac{\epsilon}{10^{-6}} \right) \quad (7)$$

where $I_o = 10^{38} \text{ kg}\cdot\text{m}^2$ is the nominal quadrupole moment of a neutron star, the gravitational radiation is emitted at frequency f_{GW} equal to twice the star's rotation frequency and r is the distance to the neutron star.

As the scaling relationship in Equation 7 indicates, only nearby neutron stars in our Galaxy will be potentially observable as continuous wave sources, and will require optimal conditions, such as highly asymmetric geometries, near-millisecond spin rates, etc. However, the history of science has shown that observations in new wave bands bring surprises, as the universe is filled with objects that were previously not anticipated by the scientists who designed the new instrumentation.

Homework 3

At 156.3 pc from Earth, PSR J0437-4715 is the nearest millisecond radio pulsar, with a rotation period of 5.76 ms. If LIGO can confidently measure a strain value near this frequency of 5×10^{-24} , and $I_{zz} \approx I_o$, what is the minimum value of ϵ that will allow a continuous gravitational wave detection from PSR J0437-4715 ?

Homework 4

A [An extended search for gravitational waves from the Crab and Vela pulsars](#) the authors used data from Virgo to place a limit of about 4 meters on the size of the “bump” that could have been present on the surface of the neutron star that is the Crab pulsar. Assuming that the radius of the Crab pulsar is 10 km, what is the ellipticity ϵ that is associated with this limit? Note: a simple way to do this problem is to approximate $\epsilon \approx \frac{\delta r}{r}$. For a more complicated method, use Equation 6, and recall that the diagonal elements for the moment of inertia tensor for a solid sphere are given by $I_{ii} = (2/5)MR_{ii}^2$.

5 Impulsive Events

Observational evidence has continued to accumulate that supernovae explosions are asymmetric (e.g., this [recent news story from NASA's NuSTAR mission](#)). Long-duration (greater than two seconds) gamma-ray bursts (GRBs), in which exploding massive stars shoot out highly energetic beams of accelerated particles as their cores collapse to form black holes, are another impulsive phenomena which may have asymmetric components. Both supernovae and GRBs are prime candidates to produce bursts of gravitational radiation, and emit roughly comparable amounts of electromagnetic energy ($\sim 10^{44}$ J). However the types of progenitors that meet such violent ends and the conditions immediately prior to detonation are varied. Numerical simulations of supernovae and gamma-ray bursts almost always assume spherical symmetry in order to simplify the computational complexity. Therefore it is not straightforward to create a catalog of hypothetical gravitational waveforms that can be matched with impulsive events or to accurately predict the strains that might be measured in these cases. Figure 13 shows an example gravitational waveform that might signify the occurrence of an impulsive event.

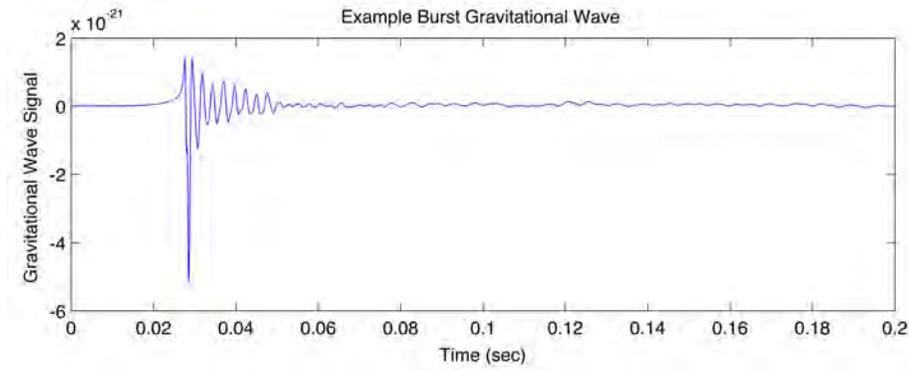


Figure 13: Example gravitational wave form from a burst-like event. The gravitational wave signal is the strain h . Credit: A. Stuver

Riles[6] posits that a detectable burst from a supernova in our Galaxy will have a strain that scales according to:

$$h \sim 6 \times 10^{-21} \left(\frac{E}{10^{-7} M_{\odot} c^2} \right)^{\frac{1}{2}} \left(\frac{1 \text{ms}}{T} \right) \left(\frac{1 \text{kHz}}{f} \right) \left(\frac{10 \text{kpc}}{r} \right) \quad (8)$$

where the duration of the burst T is scaled to 1 ms, the frequency of the radiation f is scaled to 1 kHz, and the distance to the supernova r is scaled to 10 kpc (the approximate distance to the center of our Galaxy). The nominal value for the energy in the gravitational waves is E which scales by a factor that equates to around 1.8×10^{40} J or about 2×10^{-4} of the total energy in the supernova. Unfortunately, the last known supernova to occur in our Galaxy was observed by Kepler in 1604. The closest supernova in modern times, SN 1987A, occurred in the Large Magellanic Cloud, at a distance of about 50 kpc, but predated the operation of LIGO.

In [Searching for gravitational waves associated with gamma-ray bursts detected by the InterPlanetary Network](#) the authors combined data from LIGO and Virgo detectors operating between 2005 - 2010. In this work, over 220 known gamma-ray burst occurrence times were searched for evidence of accompanying gravitational wave bursts. Due to the uncertainties in the waveform modeling, no assumptions were made as to the exact shape of the expected signals. Note that short GRBs (less than 2 seconds in duration, and about 10% of the sample) are considered by many to be associated with neutron star binary coalescence events, which were discussed earlier. From observations of the optical counterparts of long GRBs, we have learned that the average distance to these objects is about 10.6 billion light years (redshift 2.2). Short GRBs are a bit closer, with an average distance of 5 billion light years (redshift 0.5) [8]. Therefore it is not surprising that gravitational wave signals were not detected from the bursts in this sample. Similar to the “reach” statistic described above for coalescence binary signals, the limits for these results are quoted in terms of “exclusion distances,” *i.e.*, redshifts within which signals would have been expected to be detected. For this sample, the farthest distance excluded had a redshift of 10^{-2} which equates to about 43 Mpc (120 million light years).

Homework 5

Examine the exclusion distance figure in [Searching for gravitational waves associated with gamma-ray bursts detected by the InterPlanetary Network](#). The authors argue that improvements in the sensitivity of the LIGO and Virgo detectors and the range of uncertainties in modeling may allow future detections of gravitational waves from GRBs. Do you agree? Explain.

Homework 6

Soft gamma-ray repeaters (SGRs) are isolated neutron stars that unleash tremendous amounts of energy during starquakes. Also known as “magnetars”, these rare systems have megafares about once a decade, accompanied by persistent, time varying gamma-ray emission that often provides evidence of the spin rate of the underlying, highly magnetized ($\sim 10^{11}$ Tesla) neutron star.

On December 27, 2004, the brightest megafare (to date) was detected from SGR 1806-20. This magnetar is located about 50,000 light years (14.5 kiloparsecs) from Earth and the total luminosity in the megafare was about 10^{39} J. If the gravitational wave emission fraction for megafares from SGRs is similar to that of a supernova (as scaled in Equation 8), at what distance would you expect to see gravitational wave emission from an SGR? Are detections of gravitational waves from SGRs likely?

6 Stochastic Background

Cosmic background radiation has been observed across the electromagnetic spectrum. Some of this radiation, like the Cosmic Microwave Background (CMB), is truly cosmological in origin, originating from processes in the early Universe (at 300,000 years after the Big Bang), with wavelengths that have been stretched as the Universe has expanded over the past 13.7 billion years. In contrast, the observed x-ray background has been shown to consist of a combination of x-rays from nearby hot gas in our local bubble and unresolved higher-energy emission from distant active galaxies (galaxies that are dominated by accreting central super-massive black holes).

Most models of the very early Universe (10^{-32} – 10^{-35} s after the Big Bang) predict an incoherent background of cosmic gravitational waves. Like the electromagnetic waves in the CMB, these gravitational waves would have been stretched as the Universe expanded. And since they would have originated at far earlier times, they would carry valuable information about conditions in the very early Universe that are unobtainable by any other means.

Of course, it is also possible that some part of the gravitational wave background could be comprised of numerous unresolved sources, similar to the x-ray background. Whatever its source, once detected, the background gravitational waveforms would be extremely erratic, similar to what is shown in Figure 14. And by analogy to what is observed in electromagnetic astronomy, discrete sources will likely outshine any background radiation by factors of hundreds to thousands. If so, the detection of any stochastic gravitational radiation will have to wait for future generations of gravitational wave detectors, beyond

LIGO and Virgo.

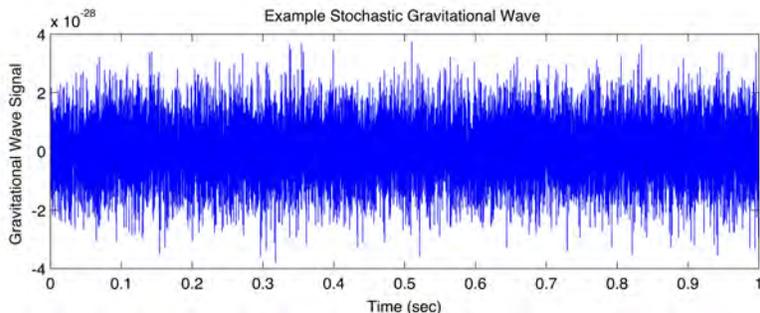


Figure 14: Example gravitational wave form from the stochastic background. The gravitational wave signal is the strain h . Credit: A. Stuver

Despite the extreme challenge of separating stochastic gravitational waves from background noise arising from environmental and instrumental effects, recent studies have searched for these elusive signals by looking at correlations in the noise between pairs of detectors. In a recent paper, [Search for a gravitational-wave background using co-located LIGO detectors](#), the authors analyzed data from two different interferometers that were both located at LIGO Hanford Observatory (LHO). Until 2009, LHO operated both a 4-km interferometer (H1) and a 2-km interferometer (H2). By comparing data between these two detectors, correlations resulting from local environmental noise sources were able to be removed, in order to look for a remaining stochastic background signal in portions of the frequency spectrum that were not contaminated with the local noise. However, there was no evidence for any stochastic background signal in this uncontaminated frequency range (460 - 1000 Hz).

Earlier work[9] cross-correlated strain data between pairs of detectors at LHO(H1 and H2) with the 4-km LIGO Livingston Observatory Detector (L1). As Figure 15 shows, the limit that was able to be set between 40 and 500 Hz, was about a factor of 100 below the strains directly measurable by the individual detectors in the two year-long observing run. Converting the strain measurements into a limit on the fraction of the total energy density in the gravitational wave background in this frequency region compared to the critical energy density, we find that $\Omega_{GW} = 6.9 \times 10^{-6}$. By comparison, the current best values of the fractional energy densities are: $\Omega_M = 0.29$ for matter and $\Omega_\Lambda = 0.71$ for dark energy. So although the stochastic gravitational wave background may someday provide valuable information about conditions in the early Universe, the waves themselves will not alter the Universe's eventual destiny.

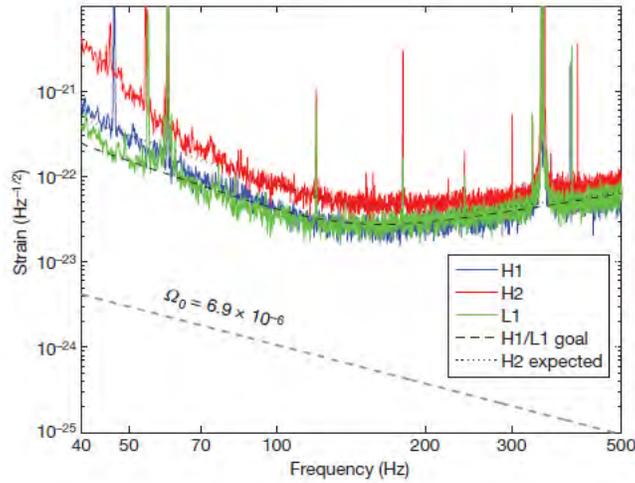


Figure 15: Limits to the gravitational wave energy density in the stochastic background compared to the strain sensitivities of the various LIGO detectors during a two-year observing run[9].

Homework 7

The critical energy density of the Universe is given by:

$$\rho_{crit} \equiv \frac{3H_o^2}{8\pi G} \quad (9)$$

where $H_o = 70.5 \text{ km/s/Mpc}$ and G is the gravitational constant. First find the value of ρ_{crit} in J m^{-3} . Then calculate the maximum number of protons per cubic meter it would take (assuming 100% of the proton's rest mass is converted into energy) to stay below the limit on the gravitational wave fraction of the critical energy density (in the LIGO band) given above of $\Omega_{GW} = 6.9 \times 10^{-6}$.

References

- [1] Sathyaprakash, B.S. and Schutz, B.F., Living Rev. Relativity, 12 (2009) 2, <http://www.livingreviews.org/lrr-2009-2>
- [2] Hulse, R.A. and Taylor, J.H., *Astroph. J.* 195 (1975) L51
- [3] Burgay, M. et al., An increased estimate of the merger rate of double neutron stars from observations of a highly relativistic system, *Nature*, 426, 531 (2003).
- [4] Hughes, Scott A., Gravitational waves from merging compact binaries, *Ann. Rev. Astron. Astrophys.* 47:107-157, 2009
- [5] Saulson, P. R., *Fundamentals of Interferometric Gravitational Wave Detectors* (World Scientific, Singapore 1994)
- [6] Riles, K., Gravitational Waves: Sources, Detectors and Searches, *Progress in Particle & Nuclear Physics* 68 (2013) 1
- [7] Abadie, J. et al., *Class. Quant. Grav.* 27 (2010) 173001
- [8] Cominsky, L., Gamma-ray Bursts, in the McGraw-Hill Encyclopedia of Science and Technology, 11th edition (2013)
- [9] B. Abbott et al. (LIGO and Virgo Collaborations), An upper limit on the stochastic gravitational-wave background of cosmological origin, *Nature*, 460, 990 (2009).